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ORBIT OF THE SPECTROSCOPIC BINARY π ARIETIS

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The binary character of π Arietis ($\alpha = 2^{\text{h}} 43^{\text{m}} 7$, $\delta = +17^{\circ} 3'$, mag. 5.30, type B5) was announced by Adams in the *Astrophysical Journal*, Vol. 35, p. 173. The present orbit is based on fifty spectrograms secured during the years 1914, 1915, 1916 and 1917.

The character of the spectrum prevents accurate determination of the orbital elements from any reasonable number of observations. The range is not very large, the lines are of poor quality, and measures of them subject to considerable uncertainty. So that the number of plates is inadequate to provide accurate values for the eccentricity and the longitude of periastron. The wave-lengths of the lines used are given in Table I. On account of their poor character, they were not adjusted to make the sum of the residuals zero. The material to do this, however, can be found in the table of measures.

TABLE I

λ	λ	λ
3933.825	4131.047	4388.100
4026.352	4143.928	4471.676
4101.890	4267.301	4481.400
4121.016	4340.634	

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TABLE H
OBSERVATIONS OF π ARIETIS

Plate	Observer*	Date	Julian Day	Velocity	Weight	Phase from J. D. 2,420,370	O-C
Mt. Wilson		1910, Dec. 24.....	2,419,030.693	- 9	1.885	-2
"		1911, Jan. 19.....	056.700	-12	0.914	0
"		Feb. 10.....	078.660	+32	3.400	-1
"		Dec. 12.....	383.625	+ 7	0.249	-6
6310	H	1914, Aug. 25.....	2,420,370.810	- 5.9	1.0	0.810	+3
6380	H	Sept. 15.....	391.795	+14.5	1.5	2.525	0
6418	C	" 21.....	397.785	-15.4	1.5	0.807	-6
6435	Y	" 27.....	403.885	+36.4	1.5	3.053	+6
6450	C	" 30.....	406.854	+ 1.8	1.5	2.168	-1
6482	H	Oct. 4.....	410.768	+ 7.2	1.0	2.228	+2
6571	C-P ¹	Nov. 23.....	460.689	- 5.9	1.0	2.047	-4
7406	Y	1915, Nov. 13.....	815.762	+ 5.2	0.5	2.552	-11
7417	H	" 17.....	819.784	+24.1	1.0	2.720	+2
7428	Y	" 24.....	826.641	- 2.8	0.5	1.869	+5
7431	H	" 25.....	827.578	+26.6	1.0	2.806	+2
7452	P	Dec. 20.....	852.524	- 5.3	1.5	0.674	-1
7457	Y	" 28.....	860.591	- 9.5	0.5	1.133	+6
7480	H	1916, Jan. 13.....	876.507	-17.1	1.0	1.633	-4
7806	Y	Sept. 9.....	2,421,116.868	+29.8	1.0	3.046	0
7825	Y	" 30.....	137.862	-21.0	1.0	0.916	-9
7839	Y	Oct. 2.....	139.855	+30.9	1.0	2.909	+3
7846	H	" 3.....	140.872	+ 9.0	1.0	0.072	-12
7850	Y	" 4.....	141.703	-10.4	1.0	0.903	+1
7860	H	" 6.....	143.744	+27.6	1.0	2.944	-1
7866	Y	" 9.....	146.661	+ 5.2	1.0	2.007	+8
7867	Y	" 9.....	146.721	+ 4.1	0.5	2.067	+5
7871	Y	" 11.....	148.627	+13.6	1.0	0.119	-5
7872	Y	" 11.....	148.680	+23.3	1.0	0.172	+6
7880	Y	" 24.....	161.726	-11.0	0.5	1.656	+2
7883	Y	" 29.....	166.687	+13.0	0.5	2.763	+1
7885	Y	" 30.....	167.681	+34.8	0.5	3.757	+9
7891	Y	Nov. 5.....	173.589	- 7.6	1.0	1.957	-3
7892	Y	" 5.....	173.640	- 9.1	1.0	2.008	-6
7895	Y	" 6.....	174.741	+23.9	0.5	3.109	-7
7900	Y	" 14.....	182.703	+25.9	1.0	3.363	-7
7904	H	" 20.....	188.567	- 8.1	0.5	1.519	+7
7906	Y	" 20.....	188.704	-18.0	1.5	1.656	-5
7910	Y	" 21.....	189.582	+20.9	1.0	2.534	+5
7911	Y	" 21.....	189.639	+21.3	1.5	2.591	+3
7948	Y	Dec. 17.....	215.516	- 9.1	1.0	1.490	+6
7949	Y	" 17.....	215.573	-15.5	1.0	1.547	-1
7953	Y	" 19.....	217.535	+40.2	1.0	3.509	+8
7974	Y	" 30.....	228.454	+22.5	1.0	2.866	-4
7975	Y	" 36.....	228.504	+17.7	1.0	2.916	-10
7987	Y	1917, Jan. 12.....	241.518	+ 0.3	1.0	0.514	-1
7988	Y	" 12.....	241.575	+12.6	1.0	0.571	+13
7994	Y	" 16.....	245.532	-10.7	1.0	0.674	-6
7996	Y	" 16.....	245.608	+ 4.2	1.0	0.750	+1
8041	Y	Feb. 11.....	271.479	+26.8	1.0	3.497	-5
8042	Y	" 11.....	271.535	+32.7	1.5	3.553	+2
8051	Y	" 12.....	272.483	- 5.0	1.5	0.645	-1
8057	Y	" 15.....	275.533	+31.1	1.0	3.697	+3
8062	Y	" 18.....	278.490	+25.7	1.5	2.800	+1
8097	Y	Mar. 6.....	294.503	+36.4	1.5	3.397	+4

* P=Plaskett; H=Harper; C=Cannon; P¹=Parker; Y=Young

MEASURES OF π ARIETIS

λ	6310		6380		6418		6435		6450		6482		6571		
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	
3933.825	—		—		—	32.6	1	+ 12.7	1	— 13.6	1	— 12.8	1	+ 0.8	1
4026.352	— 10.4	1	— 3.4	1	—	26.8	1	+ 28.6	1	— 26.0	1	— 7.8	1	+ 4.7	1
4101.890	— 26.9	1	— 13.0	1	—	40.8	1	+ 23.2	1	— 26.0	1	—			
4143.928	—		—		—	19.2	1	—		— 5.3	1	—			
4121.016	—		—		—		—		—	— 10.4	1	—			
4340.634	— 19.1	1	— 7.8	1	—	39.4	1	+ 7.8	1	+ 2.8	1	— 9.0	1	+ 10.6	1
4388.100	— 37.3	1	—		—		—		—		—		—		
4471.676	— 48.9	1	— 9.2	1	—	37.7	1	+ 13.6	1	— 23.5	1	+ 14.2	1	— 3.7	1
4481.400	— 52.3	1	— 7.4	1	—	38.7	1	—	—	— 15.0	1	— 19.9	1	+ 6.2	1
Weighted mean	— 33.60		— 8.16		— 36.00		+ 17.52		— 15.11		— 8.21		+ 3.70		
V _a	+27.82		+22.80		+20.79		+18.49		+17.30		+15.66		— 9.22		
V _d	+ 0.14		+ 0.09		+ 0.09		— 0.14		— 0.10		+ 0.04		— 0.08		
Curv.	— 0.28		— 0.28		— 0.28		— 0.28		— 0.28		— 0.28		— 0.28		
Radial Velocity	— 5.9		+ 14.5		— 15.4		+ 36.4		+ 1.8		+ 7.2		— 5.9		

MEASURES OF π ARIETIS—Continued

λ	7406		7417		7428		7431		7452		7457		7480	
	Vel.	Wt.												
3933·825									+ 12·0	1			+ 31·6	1
4026·352	+ 28·3	1			+ 44·5	1	+ 29·6	1	+ 39·6	1	+ 36·6	1		
4101·890	+ 31·7	1	+ 13·5	1			+ 26·1	1	+ 12·1	1	00·0	1		
4143·928	+ 38·7	1									+ 27·1	1		
4267·301	+ 38·5	1												
4340·634	+ 7·3	1	+ 44·6	1	- 6·2	1	+ 27·1	1	+ 34·5	1	+ 3·9	1	+ 4·5	1
4388·100	+ 1·2	1	+ 25·7	1			+ 38·0	1	+ 13·5	1				
4471·676	+ 31·0	1	- 2·5	1	+ 17·8	1	+ 37·9	1	+ 1·9	1	+ 3·8	1	- 11·2	1
4481·400	- 1·2	1	+ 26·3	1	+ 3·1	1	+ 54·4	1					+ 4·4	1
4131·047							+ 18·2	1						
Weighted mean	+9·57		+30·66		+7·05		+36·82		+16·00		+14·85		+11·31	
V_a	-4·00		-6·08		-9·57		-10·02		-21·17		-24·01		-28·14	
V_d	-0·15		-0·21		+0·01		+0·11		+0·12		-0·07		+0·01	
Curv.	-0·28		-0·38		-0·28		-0·28		-0·28		-0·28		-0·28	
Radial Velocity	+5·2		+24·1		-2·8		+26·6		-5·3		-9·5		-17·1	

MEASURES OF π ARIETIS—Continued

λ	7806		7825		7839		7846		7850		7860		7863						
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.					
3933.825	—		—	46.1	1				—	23.4	1	+ 4.3	1						
4026.352	+	13.0	1	—	19.1	1	+	28.6	1			—	10.2	1	+ 19.5	1			
4101.890	—		—	36.2	1	+	19.6	1		—	14.8	1	+ 10.2	1	+ 19.5	1			
4121.016										—	38.7	1							
4143.928	—	6.2	1																
4267.301			—	28.7	1										+ 3.2	1			
4340.634	—	11.3	1	—	46.2	1	+	19.2	1	—	19.1	1	—	14.6	1	+ 19.2	1		
4388.100	—	4.7	1	—	42.0	1	+	12.8	1	+	11.6	1	—	51.3	1	— 2.3	1		
4471.676	+	29.7	1	—	44.3	1	+	1.2	1	—	7.4	1	—	3.7	1	+ 9.8	1	+ 7.4	1
4481.400	+	13.7	1	—	32.4	1	+	8.7	1	—	10.6	1	—	33.1	1	+ 28.7	1	— 11.2	1
Weighted mean	+ 5.70		—37.70		+15.01		— 6.38		—25.67		+13.25		— 8.00						
V_a	+24.42		+17.09		+16.28		+15.82		+15.45		+14.58		+13.27						
V_d	— 0.01		— 0.11		— 0.12		— 0.13		— 0.14		+ 0.09		+ 0.20						
Curv.	— 0.28		— 0.28		— 0.28		— 0.28		— 0.28		— 0.28		— 0.28						
Radial Velocity	+29.8		—21.0		+30.0		+ 9.0		—10.4		+27.6		+ 5.2						

MEASURES OF π ARIETIS—Continued

λ	7867		7871		7872		7880		7883		7885		7891	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
3933·825													+ 7·2	$\frac{1}{4}$
4026·352	+ 5·2	$\frac{1}{2}$			+ 6·1	$\frac{1}{2}$							- 3·0	$\frac{1}{4}$
4143·928													- 5·3	$\frac{1}{4}$
4340·634			- 6·7	$\frac{1}{2}$	+ 12·4	1	- 28·2	$\frac{1}{2}$	+ 18·0	$\frac{1}{2}$	+ 32·2	$\frac{1}{2}$	- 24·3	$\frac{1}{2}$
4388·100			+ 4·6	$\frac{1}{2}$	+ 12·2	$\frac{1}{2}$			- 16·3	$\frac{1}{2}$			- 9·3	$\frac{1}{2}$
4471·676	- 14·9	$\frac{1}{2}$	- 9·8	$\frac{1}{2}$			+ 1·2	$\frac{1}{2}$	+ 4·9	$\frac{1}{2}$	+ 3·7	$\frac{1}{2}$	- 12·4	$\frac{1}{2}$
4481·400	- 3·0	$\frac{1}{2}$	+ 17·6	$\frac{1}{2}$	+ 12·6	$\frac{1}{2}$	- 23·2	$\frac{1}{2}$	+ 32·4	$\frac{1}{2}$	+ 48·6	$\frac{1}{2}$	+ 11·3	$\frac{1}{2}$
4267·301													+ 10·1	$\frac{1}{2}$
Weighted mean	- 8·95		+ 1·42		+ 11·14		- 16·73		+ 9·75		+ 32·07		- 7·37	
V_s	+ 13·24		+ 12·33		+ 12·33		+ 5·98		+ 3·45		+ 2·97		- 0·13	
V_d	- 0·10		- 0·14		- 0·14		+ 0·01		+ 0·06		+ 0·06		+ 0·20	
Curv.	- 0·28		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28	
Radial Velocity	+ 4·1		+ 13·6		+ 23·3		- 11·0		+ 13·0		+ 34·8		- 7·6	

MEASURES OF π ARIETIS—Continued

λ	7892		7895		7900		7904		7906		7910		7911	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
3933·825	+ 3·2	1	+ 35·2	1	+ 30·2	1
4026·352	+ 24·0	1
4101·890	- 1·4	1	+ 32·2	1	- 13·5	1	+ 25·7	1
4143·928	- 15·4	1	+ 38·2	1
4340·634	- 15·8	1	+ 21·4	1	- 10·2	1	+ 50·9	1	+ 31·1	1
4388·100	+ 40·8	1	+ 40·8	1
4471·676	0·0	1	+ 29·1	1	+ 34·6	1	+ 1·2	1	- 11·2	1	+ 28·8	1	+ 12·4	1
4481·400	- 11·2	1	+ 8·7	1	+ 16·3	1	- 1·2	1	- 16·3	1	+ 6·2	1	+ 31·3	1
4121·016	+ 31·7	1
Weighted mean	- 8·76		+ 25·00		+ 31·02		+ 0·00		- 9·70		+ 29·32		+ 29·97	
V_s	- 0·10		- 0·71		- 4·86		- 7·96		- 7·97		- 8·44		- 8·45	
V_d	+ 0·09		- 0·09		± 0·00		+ 0·14		- 0·09		+ 0·08		+ 0·04	
Curv.	- 0·28		- 0·23		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28	
Radial Velocity	- 9·1		+ 23·9		+ 25·9		- 8·1		- 18·0		+ 20·9		+ 21·3	

MEASURES OF π ARIETIS—Continued

λ	7948		7949		7953		7974		7975		7987		7988	
	Vel.	Wt.												
4026·352	+ 56·7	½	+ 48·8	¼
4101·890	+ 33·6	½	+ 42·0	½
4340·634	+ 9·0	½	+ 12·0	½	+ 59·5	½	+ 29·4	½	+ 49·4	½	+ 17·5	1	+ 27·7	½
4388·100	+ 23·4	½
4471·676	+ 16·2	½	- 0·6	½	+ 63·5	½	+ 65·8	½	+ 38·6	½	+ 39·2	½	+ 42·5	½
4481·400	- 3·2	½	+ 4·4	+ 63·°	½	+ 51·9	½	+ 35·1	½	+ 40·0	½	+ 51·4	½
Weighted														
mean	+11·35		+ 5·26		+62·26		+47·48		+42·73		+28·55		+40·90	
V_o	-20·32		-20·32		-21·84		-24·80		-24·80		-27·95		-27·95	
V_d	+ 0·12		+ 0·09		+ 0·09		+ 0·15		+ 0·09		± 0·00		+ 0·05	
Curv.	- 0·28		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28	
Radial														
Velocity	- 9·1		-15·3		+40·2		+22·5		+17·7		+ 0·3		+12·7	

MEASURES OF π ARIETIS—Continued

λ	7994		7996		8041		8042		8051		8057		8062	
	Vel.	Wt.												
3933·825	+ 58·3	½	+ 33·3	½
4026·352	+ 33·2	½	+ 48·4	½	+ 52·7	½	+ 64·0	½
4101·890	+ 48·5	½	+ 20·0	½	+ 69·0	½
4143·928	+ 19·8	½	+ 32·4	½
4267·301	+ 22·4	½
4340·634	+ 9·0	1	+ 30·0	½	+ 61·1	1	+ 56·6	½	+ 30·5	1	+ 51·5	½	+ 61·0	½
4388·100	+ 57·4	½	+ 62·0	½
4471·676	+ 9·3	½	+ 18·6	½	+ 51·0	1	+ 73·7	½	+ 64·0	½	+ 42·9	½
4481·400	+ 20·7	½	+ 26·3	½	+ 71·4	½	+ 73·9	½	+ 19·4	1	+ 67·6	½	+ 48·8	½
Weighted mean	+18·05		+33·07		+56·94		+62·98		+25·13		+61·03		+55·90	
V_a	-28·38		-28·40		-29·82		-29·82		-29·74		-29·42		-29·82	
V_d	- 0·07		- 0·18		- 0·09		- 0·18		- 0·10		- 0·20		- 0·14	
Curv.	- 0·28		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28		- 0·28	
Radial Velocity	-10·7		+ 4·2		+26·8		+32·7		- 5·0		+31·1		+25·7	

MEASURES OF π ARIETIS—Concluded

Table II gives the journal of observations. These were formed into thirteen normal places.

NORMAL PLACES

	Julian Day	Phase from J. D. 2,420,370	Velocity	Weight	(O-C) ₁	(O-C) ₂	O-C Final
1	2,420,370	0.121	+15.30	0.6	-5.40	-3.89	-3.47
2	370	0.613	-0.50	1.0	+1.95	+1.34	+1.59
3	370	0.765	-7.90	0.9	+1.02	-0.38	-0.27
4	370	0.954	-14.50	0.5	+0.25	-1.64	-1.59
5	371	1.519	-11.50	0.5	+3.80	+3.40	+3.74
6	371	1.648	-16.50	0.6	-3.90	-3.80	-3.42
7	371	1.990	-4.20	0.9	-1.42	-0.97	-0.36
8	372	2.171	+4.00	0.6	+0.75	+1.04	+1.55
9	372	2.552	+17.20	0.9	+1.34	+0.61	+0.76
10	372	2.795	+23.65	1.0	+0.73	-0.59	-0.69
11	373	3.008	+28.76	1.2	+0.91	-0.54	-0.81
12	373	3.436	+32.80	0.9	+0.85	+0.54	+0.41
13	373	3.635	+32.50	0.6	+2.30	+2.93	+3.05
					$\Sigma p v^2 = 47.7$	35.0	34.2

Preliminary elements were selected by trial and corrected by least-squares. The result of this solution is indicated in the residuals under headings (O-C)₁ and (O-C)₂. The reduction in $\Sigma p v^2$ is satisfactory, but on computing the residuals from the observation equations they were found to differ from those computed from the ephemeris. To show the magnitude of the changes in the elements and indicate the degree of uncertainty which attaches to them, the two sets are given below.

1st.	2nd.
$P = 3.854$ days	3.854 days
$T = \text{J.D. } 2,420,370.55$	$2,420,370.375$
$e = 0.10$	0.030
$\omega = 105^\circ$	$89^\circ.05$
$K = 25$ km.	24.60 km.
$\gamma = +7.65$ km.	$+ 8.24$ km.

The main change is in the eccentricity, which was greatly over-estimated in the preliminary elements. In proceeding to a second solution ω was put at 90 degrees, and a small change made in T to correspond. The eccentricity was made 0.05, and the solution again carried through. The normal equations using Schlesinger's notation are:

$$\begin{aligned} 10 \cdot 200r + 0 \cdot 808\kappa - 1 \cdot 249\pi + 0 \cdot 705\epsilon - 0 \cdot 652\tau &= - 1 \cdot 586 \\ + 4 \cdot 863\kappa - 0 \cdot 774\pi + 0 \cdot 299\epsilon - 0 \cdot 719\tau &= + 1 \cdot 114 \\ + 5 \cdot 337\pi + 0 \cdot 182\epsilon + 4 \cdot 765\tau &= + 0 \cdot 352 \\ + 1 \cdot 300\epsilon + 0 \cdot 209\tau &= + 0 \cdot 425 \\ + 4 \cdot 285\tau &= + 0 \cdot 190 \end{aligned}$$

whence,

$$\begin{array}{ll} \tau = - 5 \cdot 54 & dT = - 125 \\ \epsilon = + 0 \cdot 44 & de = - 008 \\ \pi = + 5 \cdot 037 & d\omega = - 11 \cdot 73 \\ \kappa = + 0 \cdot 174 & dK = + 174 \\ r = + 0 \cdot 063 & d\gamma = - 19 \end{array}$$

and the final elements with the probable errors become,

$$\begin{array}{ll} P = 3 \cdot 854 \text{ days} & \\ e = 0 \cdot 042 & \pm 025 \\ T = \text{J. D. } 2,420,370 \cdot 259 & \pm 351 \text{ day} \\ \omega = 78^\circ 27 & \pm 21^\circ 9 \\ K = 24 \cdot 77 \text{ km.} & \pm 70 \text{ km.} \\ \gamma = + 7 \cdot 81 \text{ km.} & \\ a \sin i = 1,312,000 \text{ km.} & \\ \frac{m_1^3 \sin^3 i}{(m+m_1)^2} = 0 \cdot 0061 \odot & \end{array}$$

The residuals from these elements agree with the residuals from the observation equations to the nearest tenth of a kilometre. Comparison with the residuals given by the first solution shows that little has been gained, and the differences between the last two sets of elements are less than the probable errors.

The residuals given by the individual plates are tabulated under the heading (O-C) in the table of observations. The probable error of a single observation is 3.7 km.

Dominion Observatory
Ottawa
May, 1917.

